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Numerical Analysis of Stability for Temporal Bright Solitons in a $\mathcal{PT}$-Symmetric NLDC

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Abstract: $\mathcal{PT}$-Symmetry is one of the interesting topics in quantum mechanics and optics. One of the demonstration of $\mathcal{PT}$-Symmetric effects in optics is appeared in the nonlinear directional coupler (NLDC). In the paper we numerically investigate the stability of temporal bright solitons propagate in a $\mathcal{PT}$-Symmetric NLDC by considering gain in bar and loss in cross. By using the analytical solutions of perturbed eigenfunctions and corresponding eigenvalues the stability of temporal bright solitons is studied numerically. Three perturbed eigenfunctions corresponding to the two eigenvalues are examined for stability. The results show that the two degenerate eigenfunctions are unstable while other one is stable which have important result that the eigenfunctions are equilibrium function but not stable for all cases. Stability is tested by using energy of perturbed soliton that propagate thought the length of NLDC. In addition, the behavior of solitons under instable perturbation in a $\mathcal{PT}$-Symmetric NLDC can be used to design integrated optics at Nano scales, for ultrafast all optical communication systems and logic gates.

Key words: Nano Couplers, Fibers, Nonlinear Optics, Photonics, Solitons, $\mathcal{PT}$-Symmetry

1. INTRODUCTION

Pulse transmission in the nonlinear optical waveguides and fibers is governed by the nonlinear Schrodinger (NLS) equations. NLSs are integrable equations that support soliton solutions [1]. Solitons are solitary waves with notable stability

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properties. Solitons arise in many areas of physics, optical solitons have many applications in optical fibers communications, all-optical switching systems, lasers and use to transmit digital signals over long distances [2-5]. Since they were first observed and described by Russell [6], many experiments and remarkable mathematical theories have been developed to describe and study of their properties [7,8]. Although the NLSs are integrable but when the higher order term such as third order dispersion (TOD), self-steepening, Raman gain are brought into consideration, make them to be not integrable [9]. For instance, bright solitons in $\mathcal{PT}$-Symmetric nonlinear directional couplers (NLDC) are not integrable [10] and the Hamiltonian of these systems is non- Hermitian. In 1998 Bender and Butcher found a unique remarkable phenomenon that even non-Hermitian Hamiltonians can still have completely real eigenvalue spectra if they respect parity-time (*PT*) symmetry [11]. From then on, the unique properties of $\mathcal{PT}$- symmetric systems have drawn considerable attention in both quantum mechanics and optics [12]-[16].

We study the stability of soliton in $\mathcal{PT}$- Symmetric NLDC by using soliton perturbation method. In this method, first by linearizing the nonlinear-coupled equations around a soliton solution a set of eigenvalue equations are obtained. The eigenvalue equations solve analytically. The perturbed eigenfunctions should be tested numerically to clarify the stability or instability [17]. We will use the energy and shape of the perturbed solitons for study the stability in a $\mathcal{PT}$-symmetric NLDC with more precision. NLDC at Nano scales, is recently designed by nanowires [18].

2. THEORY

A. Perturbation method

The nonlinear-coupled equations for a $\mathcal{PT}$-Symmetric NLDC with gain and loss are:

$$\begin{aligned} &iu_z + u_{\tau\tau} + 2|u|^2 u = -v + i\gamma u, \\ &iv_z + v_{\tau\tau} + 2|v|^2 v = -u - i\gamma v. \end{aligned} \tag{1}$$

Where, u and v are the normalized slowly varing amplitude at bar and cross fiber coupler waveguides, z and τ indicate the length of fiber and normalized time, respectively.

To satisfy the $\mathcal{PT}$-symmetric condition, we assume that the group velocities and the second-order dispersions in fibers are matched. We normalize the coefficients of $u_{\tau\tau}$ and $v_{\tau\tau}$ to unity, and hence, two waveguides have the same Kerr nonlinear coefficients.

In Eq.(1), the first term in the right hand side is related to coupling between the modes propagating in two fiber waveguides and γ terms stand for the gain in one fiber and loss in the other. Without loss of generality, γ can be considered to be positive which means that the gain is supposed to be in bar fiber and loss in cross one. To confirm the $\mathcal{PT}$-symmetric condition, the gain and loss coefficients must be equal [12].

Equation (1) has solitons solutions as the following:

$$\psi(\tau, z) = a\,\mathrm{sech}(a\tau)\exp\left(ia^2 z\right)$$

Now we study the propagation of bright soliton under small perturbations. In the presence of perturbations, the behavior of soliton and the soliton energy is changed thought the propagation. Let add a small perturbation to the soliton solution of Eq. (1) as following:

$$\begin{aligned} U(\tau, z) &= \psi(\tau, z) + \delta U(\tau, z), \\ V(\tau, z) &= \psi(\tau, z) + \delta V(\tau, z) \end{aligned} \tag{2}$$

Substituting Eq. (2) into Eq. (1) leads to:

$$\begin{aligned} iU_z + U_{\tau\tau} - \Omega U + 2|U|^2 U &= -\cos\theta V + i\gamma\left(U - V\right), \\ iV_z + V_{\tau\tau} - \Omega V + 2|V|^2 V &= -\cos\theta U + i\gamma\left(U - V\right). \end{aligned} \tag{3}$$

Using a set of combinations as:

$$p = \frac{\delta U + \delta V}{\sqrt{2}}, \qquad q = \frac{\delta U - \delta V}{\sqrt{2}} \tag{4}$$

In addition, according to standard perturbation analysis, expanding solutions p and q into the following forms [14]:

$$\begin{aligned} p &= \exp(\nu t)[(p_1' + ip_2')\cos\omega t + (p_1'' + ip_2'')\sin\omega t], \\ q &= \exp(\nu t)[(q_1' + iq_2')\cos\omega t + (q_1'' + iq_2'')\sin\omega t]. \end{aligned} \tag{5}$$

Where:

$$p_1 = p_1' + ip_1'' \quad , \quad p_2 = p_2' + ip_2''$$
$$q_1 = q_1' + iq_1'' \quad , \quad q_2 = q_2' + iq_2'',$$

Two eigenvalue equations obtain by substituting Eq. (2) into Eq. (3) by neglecting the higher order terms include multiple of perturbation terms:

$$\begin{aligned}(L-\cos\theta)\vec{p} + 2\gamma J\vec{q} &= \mu J\vec{p},\\ (L-\cos\theta)\vec{q} &= \mu J\vec{q}.\end{aligned} \qquad (6)$$

The operator L is determined as:

$$L = \begin{pmatrix} \frac{d^2}{d\tau^2} + \Omega - 6\psi^2 & 0 \\ 0 & \frac{d^2}{d\tau^2} + \Omega - 2\psi^2 \end{pmatrix},$$

J is a skew-symmetric matrix:

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

By solving Eq. (6), the analytic eigenfunctions and their corresponding eigenvalues are obtained as:

$$\begin{aligned} y_0 &= \mathrm{sech}(a\,\tau), & \lambda &= 0\\ y_1 &= \mathrm{sech}^2(a\tau), & \lambda &= -3\\ y_2 &= \mathrm{sech}(a\,\tau)\tanh(a\,\tau), & \lambda &= 0 \end{aligned}$$

The evolution of energies of propagating solitons in the bar and cross in a $\mathcal{PT}$-symmetric NLDC, are needed to investigate the stability of soliton.
The associated puls energies in bar and cross of a $\mathcal{PT}$-symmetric NLDC are:

$$E_u = \int |u|^2 \, d\tau, \qquad E_v = \int |v|^2 \, d\tau \qquad (7)$$

We define normalized energy in the bar and cross as the ratio of output energy to the input energy:

$$e_u = \frac{\int |u(L,\tau)|^2 \, d\tau}{\int |u(0,\tau)|^2 \, d\tau}, \qquad e_v = \frac{\int |v(L,\tau)|^2 \, d\tau}{\int |v(0,\tau)|^2 \, d\tau}. \tag{8}$$

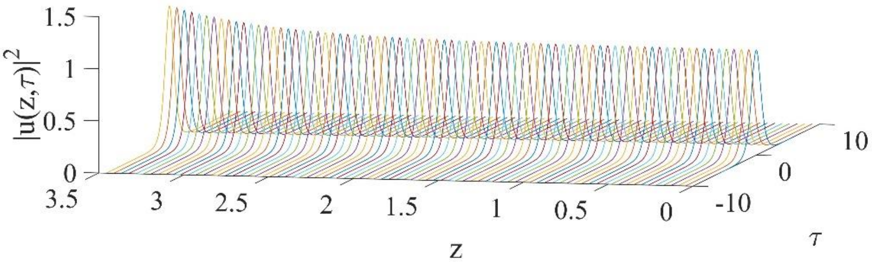
If the normalized energy is exceeded than one, gain is dominant; on the other hand, if the normalized energy were less than one, loss is dominant. If the normalized energy of the soliton is exceeded than one or less than one, the soliton is instable. In the case of normalized energy of propagated pulse in the length of NLDC equal to one the stability of pulse may be achieved. It is possible that the energy of pulse remain constant but the shape of pulse to be altered. In this case, if the shape of pulse is repeated at some interval, soliton is stable. For study the stability, we consider two cases shape and normalized energy.

3. NUMERICAL RESULTS

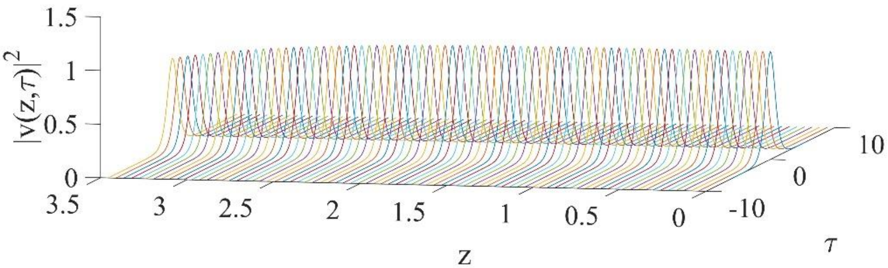
For studing the stability of soliton under the perturbation of eigenfunctions, we simulate the propagation of puturbed soliton by numerically solving Eq. (1). The initial condition is takan such that the purterbed eigenfunction plus following bright soliton is launched into the coupler:

$$u(0,\tau) = \operatorname{sech}(a\tau), \quad v(0,\tau) = \operatorname{sech}(a\tau). \tag{9}$$

The result of simulation have two output i.e. the shape and normalized energy of the perturbed soliton in the length of $\mathcal{PT}$- symmetric NLDC. In Fig.1 the pertubation eigenfunction $y_0 = \operatorname{sech}(a\,\tau)$ and its corresponding eigenvalue $\lambda = 0$ is considered. Figs. 1(a) and 1(b) show the evolution of perturded bright solitons in the bar and cross of $\mathcal{PT}$-Symmetric NLDC and in Fig. 1(c) normalized energy of pulse is plotted for bar and cross of NLDC respectively. We can clearly obsevre that perturbed bright soliton at first in bar and cross propagte without any changes but after a while in bar ampilified and attenued in cross. As we can see in Fig. 1(c) the ampilification and attenuationn in bar is continuse untile one of them is completely vanished. So these perturbed bright solitons are unstable. The usual way of thinking or expected is that the purturbed eigenfunction should be stable but the resault shows that we should be interprete the eigenfunctions as equilibrium eigenfunction not stable eigenfunction.



(a)



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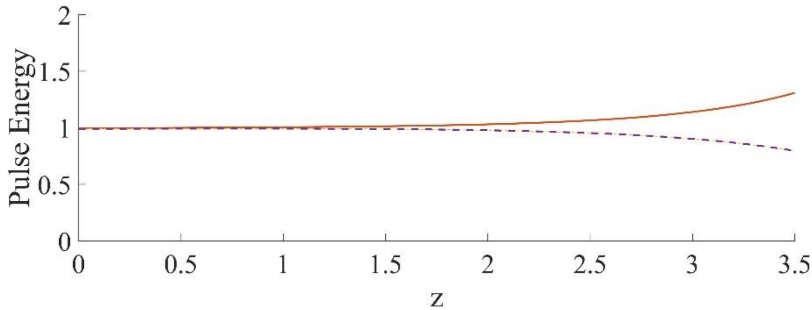
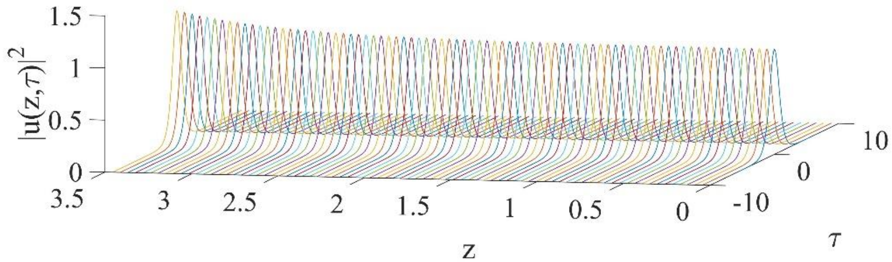


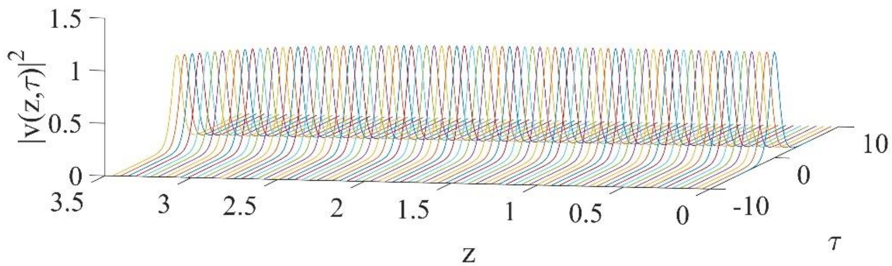
(c)

Fig. 1: Evaluation of shape and energy of perturbed bright soliton by $y_0 = \text{sech}(a\tau)$, $\lambda = 0$ (a) and (b) evolution of shape of pulse for perturbed soliton in bar and cross of a $\mathcal{PT}$- Symmetric NLDC, (c) evolution of perturbed solitons energy (solid line for bar and dashed line for cross).

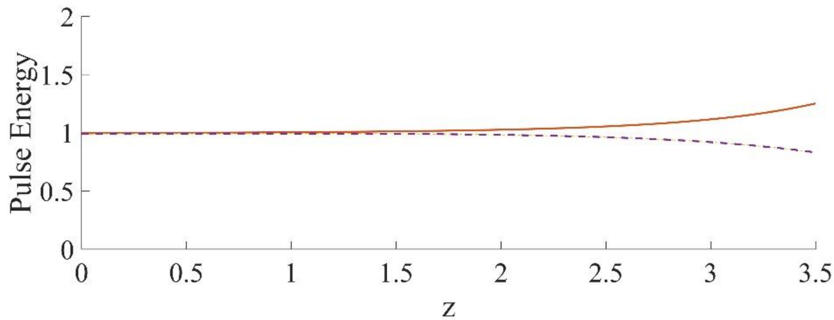
In Figs. (2) the second perturbation eigenfunction and its eigenvalue obtained in previous section, $y_1 = \text{sech}^2(a\tau)$ and $\lambda = -3$, is applied into initial bright solitons. Fig. 2(a) and 2(b) show that these perturbed solitons have the same behaviour as perturbed bright solitons in Figs. (1), bright soliton in bar is amplified and attenuation occure in cross. But the diffrence between these two figures is that in Figs. (2) this instability state happen sooner than in Figs. (1) and also as it is obvious in Fig. 2(c) amplification in bar is decresed after a while and the rate is not permenet. These perturerbed bright solitons propagating in a $\mathcal{PT}$- Symmetric NLDC with gain and loss are unstable too althogh this eigenfunction is at equilibrium but not stable.



(a)



(b)



(c)

Fig. 2: Perturbed bright soliton by $y_1 = \text{sech}^2(a\tau)$, $\lambda = -3$ (a) and (b) evolution of perturbed soliton in bar and cross of a $\mathcal{PT}$- Symmetric NLDC, (c) Evolution of perturbed solitons energy (line for bar and dash-line for cross).

In Figs. (3), the third perturbed eigenfunction, $y_2 = \text{sech}(a\tau)\tanh(a\tau)$ and its corresponding eigenvalue $\lambda = 0$ is added to the initial bright soliton. The evolution of these perturbed solitons in a $\mathcal{PT}$-Symmetric NLDC with gain and loss is examined in Fig. 3(a) and Fig. 3(b). As we can see, the stability durtion of these perturbed bright solitons is more than two others. After that the amplification happen in bar and attenuation occure in cross. Due to Fig. 3(c) evaluation of energies confirm the results in Fig. 3(a) and Fig. 3(b) and also show that the shape and energy of perturbed pulse remain constant. In this case the equilibrium eigenfunction is stable eigenfunction.

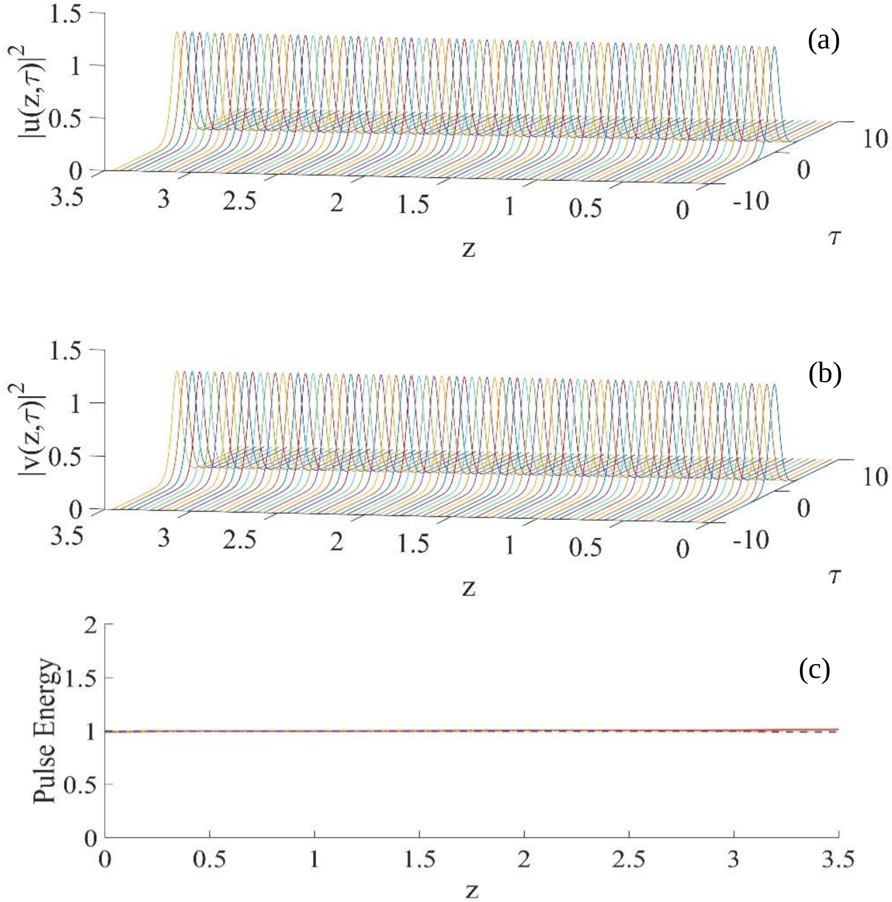


Fig. 3: Perturbed bright soliton by $y_2 = \text{sech}(a\tau)\tanh(a\tau)$, $\lambda = 0$ (a) and (b) evolution of perturbed soliton in bar and cross of a $\mathcal{PT}$- Symmetric NLDC, (c) Evolution of perturbed solitons energy (line for bar and dash-line for cross).

Numerical method show two functions are unstable but one of them is stable. Although two eigenfunctions are unstable but after impose perturbation bar is amplified while cross is attenuated, one can interpret such behavior as switching behavior which control the output with applied an input weak pulse. Such switching behavior referred to all optical switching.

Recently couplers are design by Nano wires. Nano wire couplers have very small effective area so they are efficient for designing nonlinear devices. A So a $\mathcal{PT}$-Symmetric NLDC can be designed and use for all optical switching in Nano scale.

4. CONCLUSION

In this paper we examine the stability of temporal bright solitons propagate in a $\mathcal{PT}$-symmetric NLDC with gain and loss. Three perturbed eigenfunctions and corresponding eigenvalues which have been obtained by perturbation theory are used. By applying these small perturbation to the initial bright solitons stability of them are studied numerically in two ways: 1) The evolution of perturbed temporal bright solitons and 2) evolution of their normalized energy, in a $\mathcal{PT}$-symmetric NLDC. Three perturbed eigenfunctions corresponding to the two eigenvalues are examined for stability. The results show that the two degenerate eigenfunctions are unstable while other one is stable which have important result that the eigenfunctions are equilibrium function but not stable for all cases. Stability is tested by using energy of perturbed soliton that propagate thought the length of NLDC. In addition, the behavior of solitons under instable perturbation in a $\mathcal{PT}$-Symmetric NLDC can be used to design integrated optics at Nano scales, for ultrafast all optical communication systems and logic gates.

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