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Simulation of Direct Pumping of Quantum Dots in a Quantum Dot Laser

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Abstract: In this paper, the nonlinear rate equations governing a quantum dot laser is used to simulate the transient as well as the steady-state behaviors of the laser. Computation results show that the rate equations are capable of simulating true behavior of a quantum dot laser. Then, the pump rates of the rate equations (which show indirect electrical pumping of the quantum dots through a wetting layer) are changed so that they can show direct electrical pumping of the quantum dots. Simulation results predict that a quantum dot laser with direct pumping has much lower threshold current than the indirect one. It is also shown that duration time of the transient regime to reach steady-state operation is shorter in direct pumping.

Key words: Quantum Dot Laser, Direct Pumping, Indirect Pumping, Relaxation Oscillation.

1. INTRODUCTION

Low-dimensional semiconductor structures such as quantum wells, quantum wires, and quantum dots, due to the strong quantum confinement effects, significantly change the electronic density of states compared to the bulk form of the material [1]. These changes have had many impacts in the laser technology and have led to appearance of important novel lasers such as quantum well and quantum dot lasers. Quantum dot semiconductor lasers have attracted more attention due to their high optical efficiency, high modulation bandwidth and high thermal stability in comparison with other semiconductor lasers [2-4].

Exact understanding and true simulation of the dynamic behavior of a quantum dot laser requires powerful and accurate rate equations. Most of the

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rate equations encountered in the literature are for optical pumping and include three coupled equations governing the electron-hole pair density in the wetting layer, the electron-hole pair density in the quantum dot and the photon density in the cavity [5,6]. They are only useful in the steady-state operation and are unable in the simulating and interpreting of the transient behaviors occurring in the turn-on moment. In other developed rate equations (useful for electrical pumping), the dynamics of electrons and holes are considered separately and five coupled equations governing electron and hole densities in the wetting layer, electron and hole densities in the quantum dot and photon density in the cavity are obtained. In these models the carrier-carrier scattering (which leads to the carrier capture within the quantum dots) plays an important role. A number of papers are seen in the literature which considers such scattering to be linear [7-9]. Unfortunately, such linear models are still weak for study of the transient behavior. In 2007, with a more accurate investigation of the dynamics of electrons and holes, Lüdge et al revealed that the carrier-carrier Coulomb scattering is highly nonlinear [10-12]. Their nonlinear model not only describes exactly the steady-state operation but also it is powerful enough to interpret the dynamics in the transient state. However, the pumping process in these equations is considered to be 'indirect', meaning that, the wetting layer is excited (pumped) by the pumping mechanism (here the electrical current) and the excitation is transferred from the wetting layer to the quantum dots through scattering phenomena. Generalization of these nonlinear equations to include the direct pumping of the quantum dots has not been reported in the literature. In the present paper, the nonlinear rate equations for indirect pumping formulation are changed so that they can be used for study of direct pumping.

The paper is organized as follows: In the second section, the nonlinear rate equations are reported and are changed to be useful for direct pumping. In the third section, simulation of transient and steady-state behaviors is done and the results are compared with the case of indirect pumping. Conclusions are drawn in the last section.

2. NONLINEAR RATE EQUATIONS

In the more developed models of the rate equations for quantum dot lasers, five variables are used: 1) electron surface density in the wetting layer, w_e, 2) hole surface density in the wetting layer, w_h, 3) electron surface density in the quantum dot, n_e, 4) hole surface density in the quantum dot, n_h, and 5) photon density in the cavity, n_{ph}. In the electrical pumping, electrons and holes are injected into the wetting layer with current density j. In the wetting layer, some of the charge carriers recombine through non-radiative processes and some are captured into the quantum dots by the Auger scattering. A schematic of a quantum dot laser is shown in Fig. 1, [13].

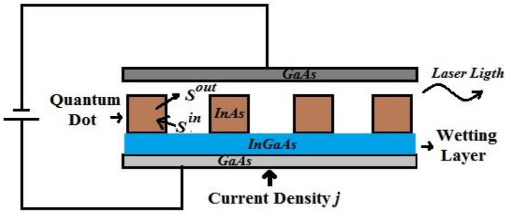

Fig. 1. Schematic of a quantum dot laser with *InAs* quantum dots in *InGaAs* wetting layer. Carriers are captured into the QDs by Auger scattering.

The captured electrons and holes within the quantum dot recombine with each other and generate light emission. This phenomenological path, as mentioned in the references [10-12], is formulated in the form of five coupled rate equations:

$$\begin{aligned}
\dot{w}_e &= \frac{j}{e_0} + \frac{n_e}{\tau_e}\frac{N^{WL}}{N^{QD}} - C_{sp} - S_e^{in} N^{WL} \\
\dot{w}_h &= \frac{j}{e_0} + \frac{n_h}{\tau_h}\frac{N^{WL}}{N^{QD}} - C_{sp} - S_h^{in} N^{WL} \\
\dot{n}_e &= -\frac{n_e}{\tau_e} - R_{sti} - R_{sp} + S_e^{in} N^{QD} \\
\dot{n}_h &= -\frac{n_h}{\tau_h} - R_{sti} - R_{sp} + S_h^{in} N^{QD} \\
\dot{n}_{ph} &= -2\kappa n_{ph} + \Gamma R_{sti} + \beta R_{sp}
\end{aligned} \tag{1}$$

where e_0 is electron charge, τ_e and τ_h are scattering relaxation times of electrons and holes, N^{WL} is density of energy states in the wetting layer, N^{QD} denotes twice of the density of quantum dots (taking into account the spin degeneration), S_e^{in} and S_h^{in} are Coulomb scattering rates of electrons and holes into the quantum dots (Auger capture).κ, Γ and β are the loss coefficient of the cavity, the optical confinement factor, and the spontaneous emission coefficient, respectively. C_{sp}, R_{sp} and R_{sti} are spontaneous recombination rate of charges in the wetting layer, spontaneous emission rate and stimulated emission rate in the quantum dots, respectively, as reported in the literature [10,11]:

$$\begin{aligned}
C_{sp} &= B_s w_e w_h \\
R_{sp} &= \frac{W}{N^{QD}} n_e n_h \\
R_{sti} &= WA\left(n_e + n_h - N^{QD}\right) n_{ph}
\end{aligned} \tag{2}$$

Where B_s is the band-band recombination coefficient, W is the Einstein coefficient and A is the area of active layer. S_e^{in} and S_h^{in} are in general nonlinear functions of the electron and hole densities in the wetting layer. In some references, the linear model is used for these dependencies [7-9] but as mentioned in the introduction such linear functionality is unable in describing the exact transient behavior of the quantum dot lasers. A nonlinear scattering model which correctly describes the transient as well as the steady-state behaviors is found in the literature [10-12]. In this model, the Coulomb scattering rates are analytically calculated as:

$$
\begin{aligned}
&S_e^{in}(w_e) = a\left(\frac{1}{1+e^{\frac{38-w_e}{5.4}}}\right)\left(\frac{e^{\frac{38-w_e}{b}}}{1+e^{\frac{38-w_e}{b}}}\right) \\
&+ c\, e^{-2(w_e-124.5)^2/29.6^2} \\
&\begin{cases} a = 0.715 + 0.6g_c - 0.19g_c^2 \\ b = -6.9 + 40.5g_c - 11g_c^2 \\ c = 0.0116 \end{cases} \\
&S_h^{in}(w_h) = \tanh(dw_h)\frac{f}{g\sqrt{\pi/2}}e^{-2(w_h-182)^2/g^2} \\
&\begin{cases} f = 8 + 0.228g_c \\ d = 0.096 - 0.0095g_c \\ g = 171 \end{cases}
\end{aligned} \tag{3}
$$

where g_c is a coefficient relating w_e and w_h by $w_h = g_c w_e$ relation and is chosen g_c=2.3 in our calculations. Details of derivation of these rates can be found in [10,11].

As seen in the Eq. (1), the j/e_0 term that plays the role of pumping rate appears only in first two equations governing wetting layer carrier densities. This means that the quantum dots are indirectly pumped by Coulomb scattering of carriers that are being injected into the wetting layer. Now, if the same term is phenomenologically added to the third and fourth equations then the direct injection of carriers and hence the direct pumping of the quantum dots is theoretically realized. In this case, Eq. (1) changes to:

$$
\begin{aligned}
\dot{w}_e &= \frac{j}{e_0} + \frac{n_e}{\tau_e}\frac{N^{WL}}{N^{QD}} - C_{sp} - S_e^{in} N^{WL} \\
\dot{w}_h &= \frac{j}{e_0} + \frac{n_h}{\tau_h}\frac{N^{WL}}{N^{QD}} - C_{sp} - S_h^{in} N^{WL} \\
\dot{n}_e &= Q\frac{j}{e_0} - \frac{n_e}{\tau_e} - R_{sti} - R_{sp} + S_e^{in} N^{QD} \\
\dot{n}_h &= Q\frac{j}{e_0} - \frac{n_h}{\tau_h} - R_{sti} - R_{sp} + S_h^{in} N^{QD} \\
\dot{n}_{ph} &= -2\kappa n_{ph} + \Gamma R_{sti} + \beta R_{sp}
\end{aligned} \tag{4}
$$

where Q is a scale factor which is related to the quantum dot geometry and must be included in order to keep the dimensionality of the rate equations. Throughout the paper $Q=1$ is assumed for simplicity. In the following section the steady-state and transient behaviors of the laser are simulated through indirect pumping, Eq. (1), and also direct pumping, Eq. (4), and the results are compared.

3. SIMULATION RESULTS

In this section, a quantum dot laser of *InAs*/*GaAs* with experimental data reported in Refs. [10,11] and shown in Table I is considered.

TABLE I

Experimental data used in the calculations [10,11].

Quantity	Value	Quantity	Value
B_s	850 $nm^2 ns^{-1}$	τ_e	5.1 ps
W	1.3 ns^{-1}	τ_h	10.8 ps
β	5×10^{-6}	N^{WL}	2×10^{13} cm^{-2}
Γ	0.0011	N^{QD}	1×10^{10} cm^{-2}
A	4×10^{-5} cm^2	κ	0.12 ps^{-1}

A. Steady-state Behavior

Rate equations for indirect pumping, Eq. (1), and direct pumping, Eq. (4), were numerically solved for steady-state behavior. To do so, the left part of all equations in the two rate equations should be let zero. Then, the steady state values of densities (i.e. w_e, w_h, n_e, n_h and n_{ph}) are calculated in terms of input current density j=0-0.8×10^4 Amp/cm^2 (for indirect pumping) and j=0-0.0008×10^4 Amp/cm^2 (for direct pumping). Simulation results are plotted in Fig. 2. Noting to the photon density, n_{ph}, plots shows that there is a threshold current for both kinds of pumping below which there is no photon population. Above the threshold n_{ph} increases linearly with pump current. The very exciting point is that the threshold current of direct pumping is nearly three orders of magnitude lower than that of indirect pimping. The other important thing is that is that, below the threshold current, the carrier densities in the wetting layer and also within the quantum dot increases almost linearly with current but above the threshold they reach their saturation values. Another important point is that, the slope of linear section of photon density plot in direct pumping is larger (nearly two times) than that of indirect pumping. The slope which is, in fact, a measure of the laser gain reveals that the gain of directly-pumped laser is larger than that of indirectly-pumped laser. Because of the fact that the high gain is due to low-loss, one can infer that by direct pumping the losses present in the wetting layer can be avoided.

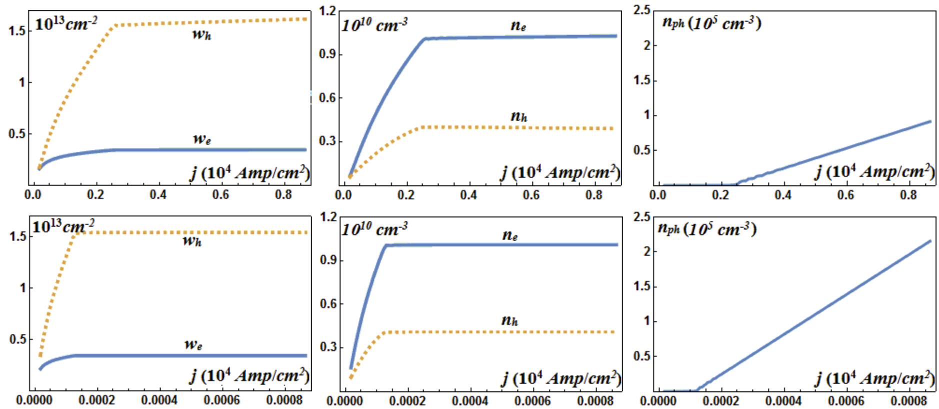

Fig. 2. Electron and hole densities in the wetting layer (left column, w_e: solid line, w_h: dashed line), electron and hole densities in the quantum dots (middle column, n_e: solid line, n_h: dashed line) and photon density in the cavity (right column) for steady-state behavior of indirect pumping (upper row from Eqs. (1)) and direct pumping (lower row from Eqs. (4)).

B. Transient Behavior

Temporal evolution of w_e, w_h, n_e, n_h and n_{ph} from turn-on moment until steady-state regime in the time range t=0-3 *ns* were calculated for indirect and direct pumping from Eq. (1) and Eq. (4), respectively, and are plotted in Fig. 3. It is clear from all plots that, after a short damping oscillatory motion beyond turn-on (called relaxation oscillation with typical frequency ≈ 3-4 *GHz* and typical duration ≈ 1-2 ns [14,15]), all densities reach to a steady-state value. Simulation of such an oscillation by linear models of rate equations is impossible. Comparison of two pumping methods yields valuable results: 1) frequency of relaxation oscillation by direct pumping is a little larger than that of indirect one. 2) The required time to reach steady-state regime by direct pumping is shorter than by indirect excitation. The shorter duration of turn-on process in direct pumping can be related to the one-stage nature of this kind of pumping. As was mentioned above, in the indirect pump, the carriers are first injected into the wetting layer and are subsequently penetrate inside the quantum dots through the scattering process. So, it is reasonable to accept that the direct pumping in which carriers are injected with no intermediate into the quantum dots must have fast response.

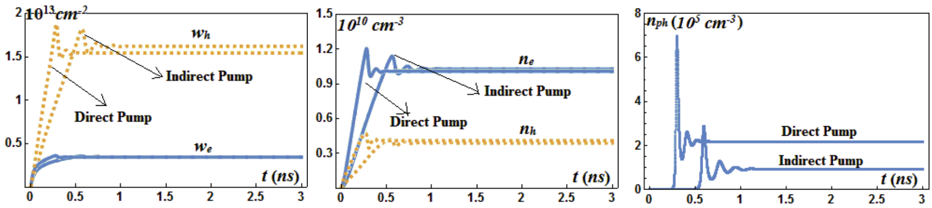

Fig. 3. Temporal evolution of electron and hole densities in the wetting layer (left plot, w_e: solid line, w_h: dashed line), electron and hole densities in the quantum dots (middle plot, n_e: solid line, n_h: dashed line) and photon density in the cavity (right plot) for indirect pumping (from Eq. (1)) and direct pumping (from Eq. (4)). Note that the above plots are for j=0.8×10^4 *Amp*/*cm*^{2} (for indirect pumping) and j=0.0008×10^4 *Amp*/*cm*^{2} (for direct pumping)

4. CONCLUSION

In this paper, the nonlinear rate equations for quantum dot lasers recently proposed by Lüdge et al [10-12] were used to simulate the laser dynamics by direct pumping of quantum dots. The pump terms in the original equations were phenomenologically changed to account for direct injection of carriers into the quantum dots. This simple assumption has interesting and important results. Simulations show that, in the steady-state regime, the threshold current of lasing for direct pumping is nearly three orders of magnitude smaller than that of indirect pumping. Other finding is that, the slope of gain regime in direct pumping is larger in comparison with indirect one which is an evidence for low-loss mechanism of direct pumping. These two important results show that if direct pumping is experimentally possible then by a very smaller current and by a larger gain the laser will operate at high powers. Regarding the transient regime, calculations show that the relaxation oscillations have larger frequency for direct pumping and, also, the time delay between turn-on moment and steady-state region is shorter in direct pumping. The latter implies that by direct pumping of the laser, its modulation bandwidth is larger than indirect pumping [16-18]. Further calculations show that in the case of direct pumping; even the pumping term in the first two terms can be neglected without any change in the output. This is due to the fact that in the direct pumping, the required current is so small that its presence or absence doesn't matter in the first two equations to trigger the indirect pumping.

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