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A Proposal for a New Method of Modeling of the Quantum Dot Semiconductor Optical Amplifiers

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Abstract: With the advancement of nanoscale semiconductor technology, semiconductor optical amplifiers are used to amplify and process all-optical signals. In this paper, with the aim of calculating the gain of quantum dot semiconductor optical amplifier (QD-SOA), two groups of rate equations and the optical signal propagating equation are used in the active layer of the device. For this purpose, the related equations are presented coherently. In our model, the rate equations that are ordinary differential equations (ODE) are solved by the Runge-Kutta method. The rate equations are based on the occupation probabilities of the energy levels instead of the carrier densities. On the other hand, the signal propagating equation is a partial differential equation (PDE) and is solved by using the SLICE technique. Therefore, a suitable solution for numerical modeling is presented. Based on the presented method, modeling is implemented in the MATLAB environment. The modeling results show a remarkable accuracy of the model. Also, the proposed model is simple and the runtime is too short in comparison with other similar models.

Keywords: Numerical Modeling, Gain, Optical Amplifier, Quantum Dot Semiconductor.

1. INTRODUCTION

The quantum dot (QD) lasers and semiconductor optical amplifiers (SOAs) have directed the attention of many researchers because QD exhibits many attractive properties that are useful for many applications [1, 2]. The unique electronic and optical properties of QD, such as sharp optical transitions, high differential gain, low threshold current, and low temperature sensitivity, can be utilized to fabricate efficient lasers and SOAs [3, 4].

In the past decade, the SOAs have gained a strong foothold among optical amplifiers in the small-scale network applications such as metro and access

WDM networks due to their compact size, broad spectrum, inerrability, and relatively low-cost [5]. In recent years, QD-SOAs have been extensively investigated by many researchers. Because of unique electronic and optical properties of QDs which are attributed to the atomic-like density of states of them, QD-SOAs exhibit superior operational characteristics that make them promising candidates for use in future all-optical networks [6]. A QD-SOA is a double heterostructure with a QD active region. In recent years, the use of QD-SOA has grown dramatically, using self-organization method with InAs/GaAs semiconductor materials in the active layer and an operating range of 1-1.3 μm.
The QD-SOA modeling enables prediction of the behavior of this amplifier in certain conditions and provides the possibility to check the main characteristics of the amplifier's performance. In order to analyze dynamic characteristics of the QD-SOA, it is necessary to solve two coupled equations, which are the rate equations and the wave propagation equation in the active layer of the device.

Although there are many published models and numerical studies for QD-SOAs, most of these models give a complicated analysis of the equations that make the numerical method more complicated with long computational time [7-10]. The rate equations that describe the carrier dynamics of the QD-SOA are many coupled equations that are usually solved numerically by the Rung-Kutta method [11, 12]. Rung-Kutta method is a precise integration technique that can solve a set of the ODE by choosing the proper time intervals. On the other hand, the wave propagation equation in the QD-SOA is a PDE, that analytic solutions of such equations are possible only in a particular condition and in general it is not possible to solve it by the Rung-Kutta method alone [13]. Therefore, the use of other numerical methods to solve this equation is necessary. These methods are divided into two groups, which are Pseudo and finite difference methods. Although such methods have a wide application for the analysis of wave propagation in optical fiber, these methods lose their performance for the QD-SOA due to the inseparability of the real and imaginary parts of the propagation equation. Also, the runtime of these models is too long and needs special attention to attain convergence. Another challenge for the gain modeling of QD-SOA is the lack of coherent and simple equations. In recent years, efforts have been made to give a simple and accurate model for the QD-SOA and are still ongoing.

This paper presents a coherent equations for the gain modeling of the QD-SOA. The model includes the carrier rate equations that describe the dynamic behavior of the electrons and holes. The rate equations are based on occupation probability of energy levels in the conduction and valence bands of QD active layer. Our model also includes the equation of optical signal propagation in the active region, which requires numerical solving of this equation to calculate the gain of QD-SOA. We solve these equations numerically using the SLICE technique. In our knowledge, the slice technique is not used for QD-SOA, yet. In this technique, the length of the QD-SOA is broken up into small slice, resulting in separate spatial steps. Then, using a new proposed algorithm, we solve the equations of the rate and wave propagation numerically and obtain the QD-SOA gain parameter.

The rest of this paper is organized as follows. Section 2 describes the investigated QD-SOA structure. In section 3, the governing equations of the QD-SOA are given. In section 4, proposed model and numerical solving algorithms of equations for calculating gain are presented. Section 5 covers the modeling results obtained for the gain of QD-SOA using the proposed model. Finally, we conclude this paper in section 6.

2. INVESTIGATED QD-SOA STRUCTURE

The study of existing theories about the nature of light in semiconductors and the study of the density of the state of nanostructures shows the theoretical superiority of QDs in terms of the intense carriers confinement in the active region and the optical mode confinement.

Figure 1 shows the cross-sectional schematic of a type of QD-SOA. As seen from the figure, cubic QDs are located within an active layer, and this layer is surrounded by two coating layers. Indeed, the QD-SOA structure is similar to the optical amplifier with the double heterostructure.

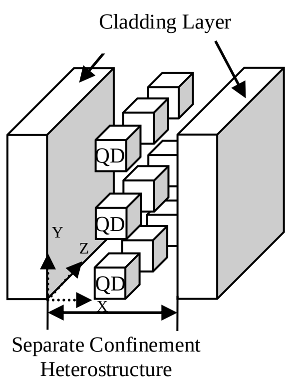

Fig. 1. Typical QD-SOA cross-sectional schematic [14].

The QD-SOA features depend on carrier transport between the QD semiconductor energy levels. When the carriers are injected to the device, they are excited to the excited stats (ES) in the active layer, quantum dots, and the wetting layer (WL). Then they will relax to the ground state (GS). In each of the active layer, wetting layers, and quantum dots, part of the carriers take part in the radiative recombination and hence the optical gain is obtained [12].

The QD-SOA structure considered in this work is shown in Fig. 2. For this optical amplifier, it is assumed that the input facet is located at the beginning of the Z-axis and the output facet is in Z = L, where L is the QD-SOA length. The investigated QD-SOA structure consist of the n-type GaAs substrate, which covered with n-type $In_{0.17}Ga_{0.83}As$ as the cladding layer, an active region containing InAs dots on a GaAs substrate with a thickness of 30 nm, a cladding layer of p-type $In_{0.17}Ga_{0.83}As$, respectively [14, 15].

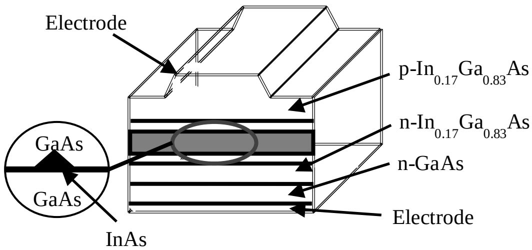

Fig. 2. Investigated QD-SOA structure.

Fig. 3 shows the energy band diagram of the active layer which is composed of the QDs. There are two types of energy states that include the ground state and excited states. We consider three excited states in the conduction band (CB) and ten excited states in the valence band (VB). The energy level associated with WL is also considered an excited state. The energy of separation between states in the CB of 60 meV and the same energy in the VB of 10 meV is considered. Also, the inhomogeneous broadening effect is considered 30 meV for dots [16].

3. EQUATIONS GOVERNING THE AMPLIFIER

One of the most important aspects of QD-SOA performance is its optical transition behavior. In fact, the behavior of optical transitions and the dynamics of carriers determine the amplifier's gain. In order to describe these transitions, some mathematical tools called the rate equations are needed to determine time variations of the carrier concentration due to current injection and time

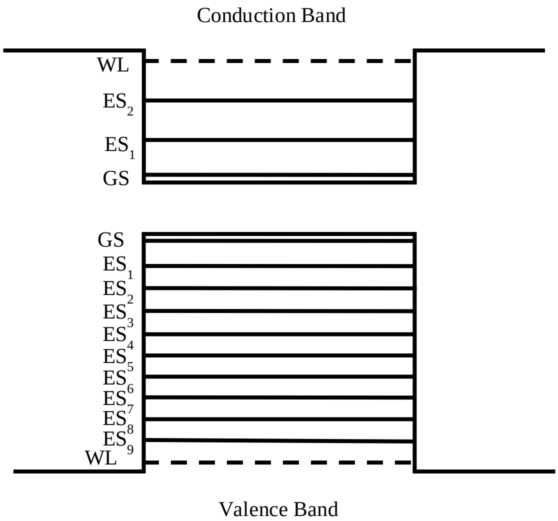

Fig. 3. QD-SOA energy band diagram.

variations photon density due to stimulated emission. For the QD-SOA, the rate equation for the WL based on the occupation probability of the levels is [17]:

$$\frac{df_w^k}{dt} = \frac{J}{edN_Q} - \frac{f_w^k(1-f_M^k)}{\tau_{wM}^k} + \frac{(1-f_w^k)f_M^k}{\tau_{Mw}^k} - \frac{f_w^k}{\tau_{wR}^k} \tag{1}$$

And for the ES is:

$$\frac{df_i^k}{dt} = \frac{f_{i+1}^k(1-f_i^k)}{\tau_{i+1,i}^k} - \frac{(1-f_{i+1}^k)f_i^k}{\tau_{i,i+1}^k} - \frac{(1-f_{i-1}^k)f_i^k}{\tau_{i,i-1}^k} + \frac{f_{i-1}^k(1-f_i^k)}{\tau_{i-1,i}^k} - \frac{f_i^k}{\tau_{iR}^k} \tag{2}$$

And for the GS:

$$\frac{df_0^k}{dt} = \frac{(1-f_0^k)f_1^k}{\tau_{10}^k} - \frac{f_0^k(1-f_1^k)}{\tau_{01}^k} - \frac{f_0^n f_0^p}{\tau_{0R}^k} - a_0(f_0^n + f_0^p - 1)S_{ph} \tag{3}$$

Which i attributes the integer to each of the levels:

$$i = 0,1,\ldots,M_k \tag{4}$$

If $k = n$, it shows the quantity dependence on the electrons of CB and if $k = p$, it shows the quantity dependence on the holes of VB (as described, in this study $n = 2$ and $p = 9$). The definition of other quantities of equations (1) to (3) is given in Table I. In the rate equations, J, the applied current density is:

$$J = \frac{I}{L.w} \tag{5}$$

Where I is the injection current to the QD-SOA and w is the waveguide width. S_{ph}, the photon density is [18]:

$$S_{ph} = |A|^2 \tag{6}$$

Where $A\,(t, z)$ is the optical signal propagated in the waveguide:

$$\frac{dA}{dz} + \frac{1}{v_g}\frac{dA}{dt} = \frac{A}{2}\left[-\alpha_l + (1 - i\alpha_H).\Gamma.g_{mat}\right] \tag{7}$$

Where α_H, linewidth enhancement factor, g_{mat}, active region gain, α_l, waveguide loss, Γ, confinement factor and v_g is the group velocity.

Since only parts of the optical mode are overlapping with the active region, the modal gain must be defined. The relationship between the active region gain and the modal gain is given by [15]:

$$g = \Gamma.g_{mat} \tag{8}$$

Where g is the QD-SOA modal gain. Moreover, the active region gain based on the occupation probability of the levels is given as [19]:

$$g_{mat} = g_{max}(f_0^n + f_0^p - 1) \tag{9}$$

Where g_{max} is maximum gain [19]:

$$g_{max} = g_0\left(\frac{\hbar\omega_0}{\hbar\omega}\right).Exp\left(-\frac{1}{2}\left(\frac{\hbar\omega - \hbar\omega_0}{\sigma}\right)^2\right) \tag{10}$$

Where $\hbar\omega$ and $\hbar\omega_0$ are the energy of the photon emitted from the QDs and input signal photon energy, respectively. σ is the inhomogeneous broadening parameter.

TABLE I
DEFINITION OF THE PHYSICAL QUANTITIES OF EQUATIONS (1) TO (3)

Symbol	Quantity
f_w^k	WL occupation probability by carriers
f_i^k	i-th state occupation probability by carriers
f_1^k	1-th state occupation probability by carriers
f_0^k	GS occupation probability by carriers
f_0^n	CB occupation probability by carriers
f_0^p	VB occupation probability by carriers
a_0	Differential gain
τ_{wM}^k	Carrier Relaxation life time from WL to M-th ES
τ_{Mw}^k	Carrier escape lifetime from M-th Es to WL
τ_{wR}^k	Carrier spontaneous emission lifetime in WL
$\tau_{i+1,i}^k$	Carrier Relaxation life time from (i+1)-th state to i-th state
$\tau_{i,i+1}^k$	Carrier Relaxation life time from i-th state to (i+1)-th state
τ_{iR}^k	Carrier spontaneous emission lifetime from i-th state
τ_{01}^k	Carrier escape lifetime from Gs to first Es
τ_{10}^k	Carrier relaxation lifetime from first Es to GS
τ_{0R}^k	Carrier spontaneous emission lifetime in GS

4. NUMERICAL MODELING

In order to model the dynamics of the carriers, the set of equations (1) to (3) should be solved numerically. From Equations (1) to (3), fifteen coupled ODE equations are obtained corresponding to all energy levels.

On the other hand, the wave propagation equation should be solved simultaneously. It should be noted that the relationship of the optical signal propagated in the waveguide can also be expressed [13]:

$$\mathbf{A} = \sqrt{\mathbf{P}}\mathbf{e}^{\mathbf{i}\phi} \tag{11}$$

Which is *P (t, z)* is the amplitude of the signal propagated in the waveguide, and *φ (t, z)* is the phase of the signal propagated inside the waveguide. Therefore, according to (6) and (11), the photon density in equation (3) is given as:

$$S_{ph} = P \tag{12}$$

By solving equation (7), which is the first order PDE, the amplitude and phase of the optical signal propagated in the waveguide of the QD-SOA in the z-propagation direction and in the time steps are obtained. Using the change of the variable of $\tau = t - \frac{z}{v_g}$, equation (7) is simplified as follows [13]:

$$\frac{dP}{dz} = (g - \alpha_l)P \tag{13}$$

$$\frac{d\varphi}{dz} = -\frac{1}{2}\alpha_H g \tag{14}$$

Since we used the slice technique in this modeling, the phase changes is neglected and equation (7) can be replaced by equation (13).

The solution of these coupled equations, which consists of spatial and temporal dimensions, is complicated. Teherefore, for numerical solution of the set of coupled equations, we use a new algorithm. As shown in Fig. 4, we divide the QD-SOA length into the equal sections. First by applying the initial values for the occupation probability of each level for the first section of the device, the active region gain, the optical signal amplitude, and the photon density as a function of time for the first section are calculated. Then, by solving 15 ODEs with fourth order Rung-Kutta method, the occupation probabilities of each level for the second section are estimated as a function of the time. Now, with the known the occupation probability of the levels, the active region gain, the optical signal amplitude, and the photon density are calculated for the second section of the QD-SOA. In order to evaluate the spatial domain, solving equation (13) in spatial dimension is necessary at the final time. By breaking the time dimension and integrating a space from the equation, Equation (13) is thus replaced [13]:

$$P(z) = P(0).Exp((g - \alpha_l)z) \tag{15}$$

Therefore, to evaluate the spatial dimension of optical amplitude, we replace active region gain of the second part in (15), and this relationship is also estimated as a function of time. The value obtained at the last moment of the optical amplitude will be in the third section of the QD-SOA. Thus, the amplitude effect is calculated in each section of the QD-SOA and is considered as the input amplitude of the next section. It should be noted that the input signal to the amplifier is a Gaussian signal that its range is:

$$P(0,t) = P_{max}.Exp(-\frac{t^2}{2\delta^2}) \tag{16}$$

The P_{max} is the amplifier saturation power. δ is the input pulse width.

Finally, by calculating the amplitude of the signal in the last section of the QD-SOA, G, the gain of the QD-SOA is calculated as follows:

$$G = \frac{P_{out}}{P_{in}} \tag{17}$$

Which is $P_{in} = P\ (Z = 0)$ and $P_{out} = P\ (Z = L)$ are input optical amplitude and the output optical amplitude, respectively. The values of the physical quantities used in this modeling are given in Table II. In Figure 5, the gain modeling algorithm of investigated QD-SOA is presented.

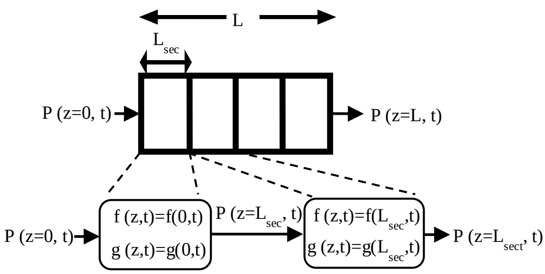

Fig. 4. Numerical Modeling Process of QD-SOA.

TABLE II
Values OF Modeling Quantities

QUANTITY	Value
a_l	10 m^{-1}
δ	20 ps
$\tau_{i+1,i}^{p}$	0.92 ps
$\tau_{i+1,i}^{p}$	0.6 ps
$\tau_{i+1,i}^{k,A}$	0.1 ps
τ_{iR}	0.2 ps
a_0	5.6×10^{-21} m^2
P_{max}	3 Watt

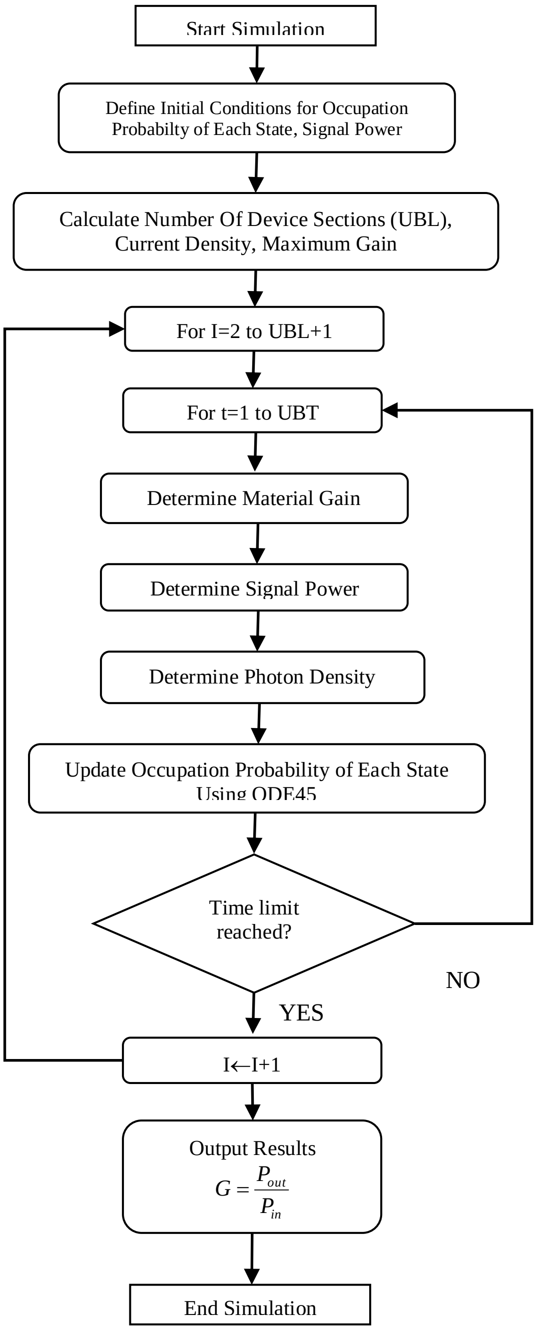
Start Simulation
Define Initial Conditions for Occupation Probabilty of Each State, Signal Power
Calculate Number Of Device Sections (UBL), Current Density, Maximum Gain
For I=2 to UBL+1
For t=1 to UBT
Determine Material Gain
Determine Signal Power
Determine Photon Density
Update Occupation Probability of Each State Using ODE45
Time limit reached?
NO
YES
I←I+1
Output Results
$G = \frac{P_{out}}{P_{in}}$
End Simulation

Fig. 5. Gain Modeling Flowchart of QA-SOA.

5. Modeling Results

QD-SOA modeling is implemented with the MATLAB software. The results of modeling for $\hbar\omega=\hbar\omega_0$ are presented in this section.

In Fig. 6, the characteristic of the active region gain as a function of the time, taking into account the maximum gain (g_{max}), is observed as a variable quantity. Over time, the active region gain changes with the change in the maximum gain. According to the theory of QD semiconductors, it is expected that in the first moments, the absorption process is dominant and the active region gain of the QD-SOA has a negative value, and over time, the number of carriers in the excited levels are increased. Hence, the stimulated emission becomes the dominant process, and as a result, the active region gain is positive.

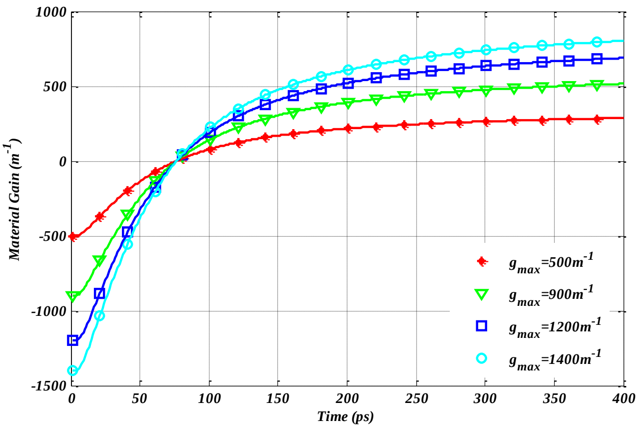

Fig. 6. The characteristic of the active region as a function of time, taking into account the maximum gain (g_{max}) as a variable quantity.

Fig. 7 shows the optical amplitude profile as a function of the time, taking into account the maximum gain (g_{max}) as a variable quantity. By increasing maximum gain, modal gain increases because of population inversion, and active region gain increase that has the direct relation to optical amplitude. Therefore, increasing maximum gain causes increasing output optical amplitude, as shown in the figure. This profile is consistent with the results of the analytical model presented in [10]. Furthermore, to check the validity of our model, we tested the model for different input values and obtained the gain of the QD-SOA. These results were compared with the experimental work values reported in [17]. For example, the QD-SOA gain for L = 4 mm, Γ = 0.4, I = 0.7 and d = 9 nm, and g_{max} = 1400 m^{-1} is 16.6 dB, which is the same as the value given in [17].

Figure 8 shows the optical amplitude characteristic as a function of the time, taking into account the normalized injected current (I) to the QD-SOA as a variable quantity. By increasing normalized injected current, stimulated emission increases and hence the gain of the QD-SOA increases. Therefore, it causes output optical amplitude increases, as shown in the figure. In this case, the QD-SOA gain for I = 0.8 and I = 0.9 are 19.7 dB and 22 dB, respectively, which matched with the reported experimental results in [17].

It should be noted that using our proposed method, it is solved just 15 equations that have acceptable accuracy compared to other papers with a large number of equations. Also, the computation time of this model lasted about 5 minutes in MATLAB environment, which is a much shorter than other reported models.

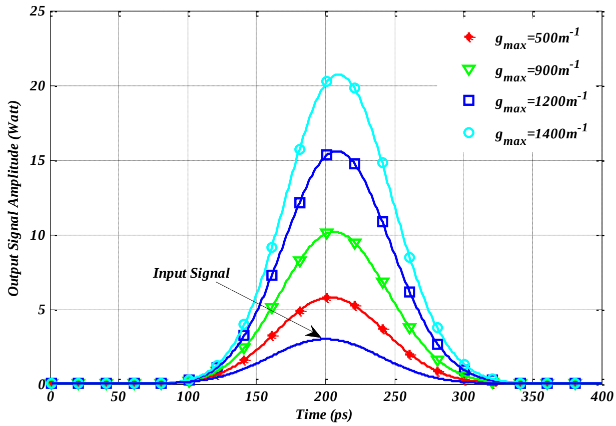

Fig. 7. The characteristic of the output signal domain as a function of the time, taking into account the maximum gain (g_{max}) as a variable quantity.

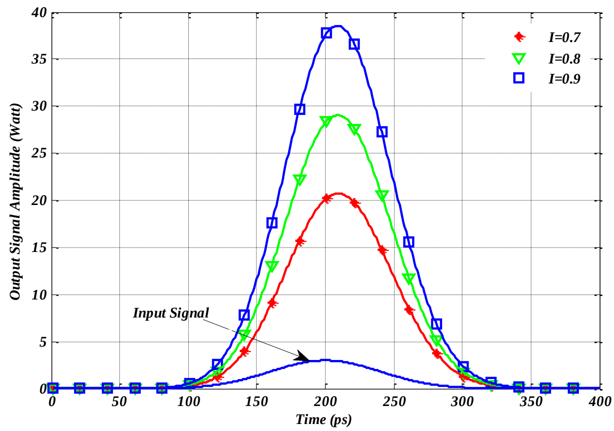

Fig. 8. The characteristic of the output signal domain as a function of the time, taking into account the normalized injected current (*I*) as a variable quantity.

6. CONCLUSION

In this paper, a simple, fast, and accurate model for QD-SOAs is proposed. the model consists of the rate equations that describe the dynamics of carriers. The number of rate equations, which are based on the occupation probability of energy levels, depends on the sum of the number of GS and ESs. The model also includes a wave propagation equation. Moreover, for numerical solution of the coupled equations, we presented a new algorithm based on the slice technique. The modeling results show remarkable accuracy comparisons of analytical and experimental data. Also, because of using fewer equations, the model has a short runtime compared with similar models.

Therefore, the proposed model and the presented algorithm for solving equations are very suitable for many applications such as computer-aided-design of QD-SOAs.

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